Standard Equation of a Parabola with vertex (h,k) and directrix $y=k-p$ is

$(x-h)^2 = 4p(y-k)$. Vertical axis of symmetry

Standard Equation of a Parabola with vertex (h,k) and directrix $x=h-p$ is

$(y-k)^2 = 4p(x-h)$. Horizontal axis of symmetry

The focus lies on the axis *p* units (directed distance) from the vertex. The coordinates of the focus are as follows:

$(h, k+p)$ Vertical axis of symmetry

$(h+p, k)$ Horizontal axis of symmetry

A **focal chord** is a line segment which passes through the focus of a parabola and has endpoints on the parabola.

The specific focal chord perpendicular to the axis of the parabola is the **latus rectum**.

A surface is considered **reflective** if the tangent line at any point on the surface makes equal angles with an incoming ray and the resulting outgoing ray. The angle corresponding to the incoming ray is the **angle of incidence** and the angle corresponding to the outgoing ray is the **angle of reflection**.

Let *P* be a point on a parabola. The tangent line makes equal angles with the following two lines:

1. The line passing through *P* and the focus.
2. The line passing through *P* parallel to the axis of the parabola.

1. Find the vertex, focus, and directrix of the parabola and sketch its graph.

a. $x^2 + 8y = 0$

$(x-0)^2 = -8(y-0)$

$V = (h,k) = (0,0)$

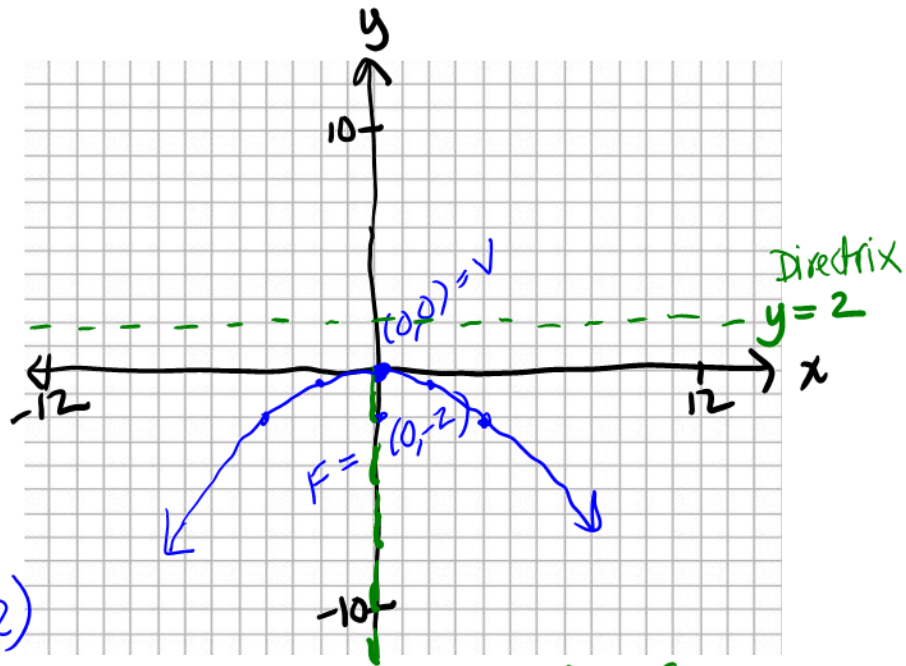
$4p = -8$

$p = -2$

$F = (0, 0 + (-2)) = (0,-2)$

Directrix: $y = 0 - (-2)$

$y = 2$



$x=0$ is axis of symm.

extra points: $x^2 + 8y = 0 \rightarrow y = -\frac{1}{8}x^2$

$y(2) = -\frac{1}{2} = y(-2)$

$y(4) = -2 = y(-4)$

b. $y^2 + 4y + 8x - 12 = 0$

$(y^2 + 4y + (+2)^2) = -8x + 12 + 4$

$(y+2)^2 = -8(x-2)$

$[y-(-2)]^2 = -8(x-2)$

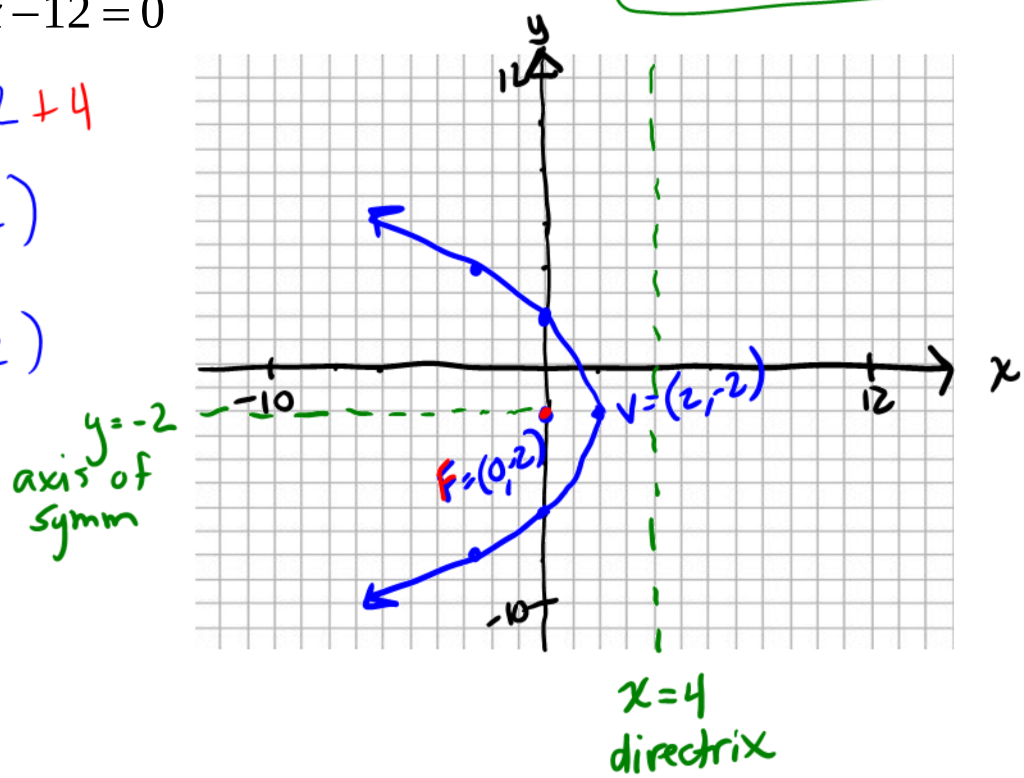
$h=2, k=-2$

$V = (2,-2)$

$4p = -8$

$p = -2$

open leftward



Standard Equation of an Ellipse with center (h,k) and major and minor axes of length 2*a and* 2*b* where $a > b,$ is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Major axis is horizontal

Or

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Major axis is vertical

The foci lies on the **major** axis, *c* units from the center, with $c^2=a^2-b^2.$

Reflective Property of an Ellipse: Let *P* be a point on an ellipse. The tangent line to an ellipse at point *P* makes equal angles with the lines through *P* and the foci.

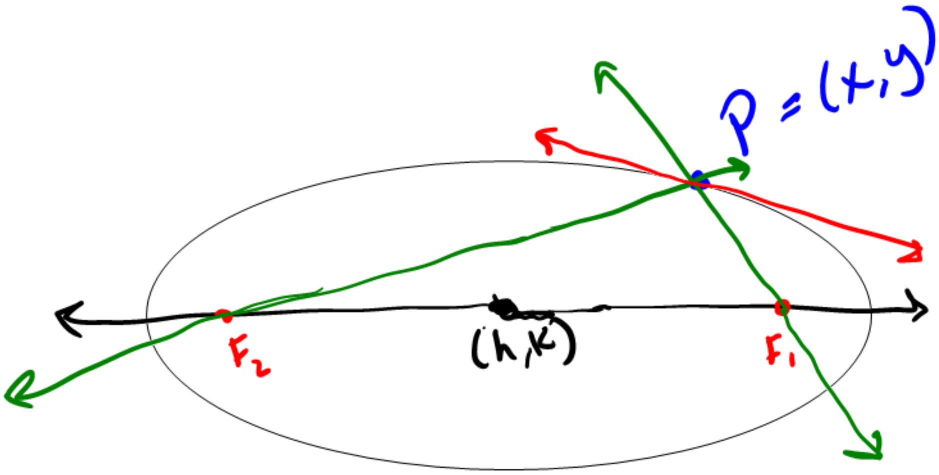
Eccentricity of an Ellipse: The eccentricity *e* of an ellipse is given by

$$e=\frac{c}{a}.$$

This measures the ovalness of an ellipse. When the eccentricity is very small and the foci are close to the center, the ellipse is nearly circular. When the eccentricity is close to one and the foci are close to the vertices, the elliple will be elongated.

2. Find the center, foci, and vertices of the ellipse, and sketch its graph.

$$16x^2+25y^2-64x+150y+279=0$$



2. Find the center, foci, and vertices of the ellipse, and sketch its graph.

$$16x^2 + 25y^2 - 64x + 150y + 279 = 0$$

$$(16x^2 - 64x \qquad) + (25y^2 + 150y \qquad) = -279$$

$$16(x^2 - 4x + (-2)^2) + 25(y^2 + 6y + (+3)^2) = -279 + 64 + 225$$

$$\frac{16(x-2)^2}{10} + \frac{25(y+3)^2}{10} = \frac{10}{10}$$

$$\frac{(x-2)^2}{\left(\sqrt{\frac{5}{8}}\right)^2} + \frac{(y-(-3))^2}{\left(\sqrt{\frac{2}{5}}\right)^2} = 1$$

$$\sqrt{\frac{5}{8}} \approx 0.79$$

$$\sqrt{\frac{2}{5}} \approx 0.63$$

$$a = \sqrt{\frac{5}{8}} \approx 0.79$$

$$b = \sqrt{\frac{2}{5}} \approx 0.63$$

$$c^2 = a^2 - b^2$$

$$c^2 = \frac{5}{8} - \frac{2}{5}$$

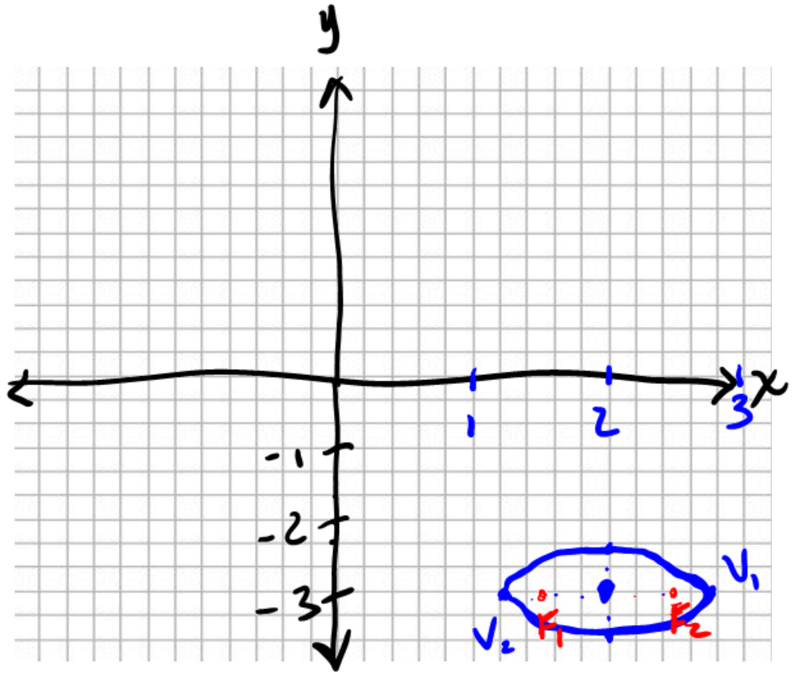
$$c \doteq 0.47$$

center: $(2,-3)$

$a \approx 0.79$

$b \approx 0.63$

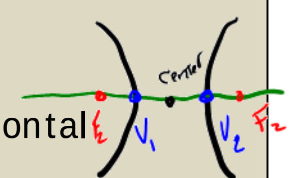
$c \approx 0.47$



Standard Equation of a Hyperbola with center (h,k) is

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

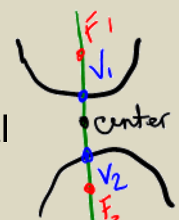
Transverse axis is horizontal



Or

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Transverse axis is vertical



The vertices are *a* units from the center. The foci are *c* units from the center, with $c^2=a^2+b^2$.

Asymptotes of a Hyperbola:

For a **horizontal** transverse axis, the equations of the asymptotes are

$$y=k+\frac{b}{a}(x-h) \quad \text{and} \quad y=k-\frac{b}{a}(x-h).$$

For a **vertical** transverse axis, the equations of the asymptotes are

$$y = k + \frac{a}{b}(x-h) \quad \text{and} \quad y = k - \frac{a}{b}(x-h).$$

The line segment of length 2*b* joining the points which are *b* units away from the center is referred to as the **conjugate axis**. The **eccentricity *e*** is $e = c/a$.

3. Find the center, foci, and vertices of the hyperbola and sketch its graph using asymptotes as an aid.

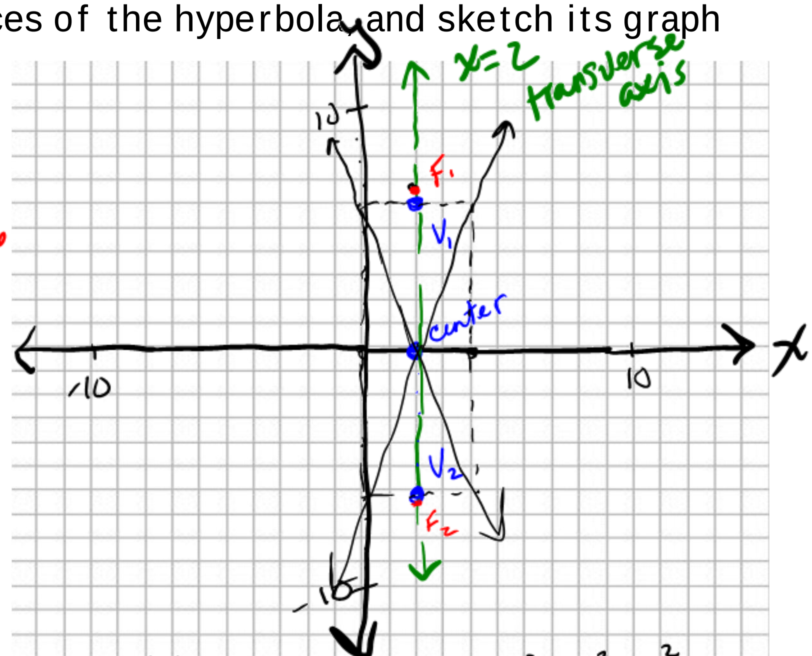
$$y^2 - 9x^2 + 36x - 72 = 0$$

$(y-0)^2 - 9(x^2 - 4x + (-2)^2) = 72 - 36$

$\frac{(y-0)^2}{36} - \frac{\cancel{9}(x-2)^2}{\cancel{36}\,4} = \frac{36}{36}$

$\frac{(y-0)^2}{(6)^2} - \frac{(x-2)^2}{(2)^2} = 1$

center: $(2,0)$



$m = \pm\frac{6}{2} = \pm 3$

$c^2 = a^2 + b^2$

$c = \sqrt{40} \approx 6.3$

4. Find an equation of the conic section.

~~a. An ellipse with vertices $(0,2)$ and $(4,2)$ and eccentricity ½.~~

$m_1 = 3;\ (2,0) \qquad m_2 = -3;\ (2,0)$

$y - y_1 = m_1(x - x_1) \qquad y - y_1 = m_2(x - x_1)$

$y - 0 = 3(x-2) \qquad y - 0 = -3(x-2)$

$\boxed{y = 3(x-2) \text{ and } y = -3(x-2)}$

b. A parabola with focus $(2,2)$ and directrix $x = -2$.

c. A hyperbola with vertices $(0,\pm 3)$ and asymptotes: $y=\pm 3x$.

5. Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

a. $12x^2-2x+12y^2=15$

b. $x+y^2-5y=1$

c. $-x^2-2x+3y^2=5y^2-2$

d. $65x^2=y^2+1$

e. $2x^2-8x+y^2+5y=6$

6. Find an equation for (a) the tangent lines and (b) normal lines to the hyperbola $\frac{d}{dx}\left(\frac{y^2}{4}-\frac{x^2}{2}\right)=\frac{d}{dx}1$ at $x=4$.

$$\frac{d}{dx}\left(\frac{(y)^2}{4}\right)-\frac{d}{dx}\left(\frac{x^2}{2}\right)=0$$

$$\frac{\cancel{2}(y)^1\cdot\frac{dy}{dx}}{2\ \cancel{4}}-\frac{2x}{2}=0$$

$$\frac{1}{2}\cdot y\frac{dy}{dx}-x=0$$

$$y'(x,y)=\frac{2x}{y}$$

$$y'(4,y)=\frac{2\cdot 4}{y}$$

$$\frac{y^2}{4}-\frac{(4)^2}{2}=1$$

$$\frac{y^2}{4}=9$$

$$y^2=36$$

$$y=\pm 6$$

at $(4,-6)$

$$y'=\frac{8}{-6}=-\frac{4}{3}$$

$$\boxed{y+6=-\frac{4}{3}(x-4)}$$

at $(4,6)$:

tangent lines

$y' = \frac{8}{6} = \frac{4}{3}$

$\boxed{y - 6 = \frac{4}{3}(x-4)}$

7. Find the point on the graph of $x^2 = 8y$ that is closest to the focus of the parabola.

8. Find the arc length of the parabola $4x - y^2 = 0$ over the interval $0 \le y \le 4$.